A Meta-Musical Evening

PETRA CINI

RAF BOCKLANDT

Splendor Amsterdam, 25 March 2023

Preface

A lot of composers have used mathematics in their work, but what happens when a composer decides to represent the meaning of the objects of mathematics, and the sensations that can be found in them, instead of focusing on their direct application or translation? The theoretical and artistic work of Petra Cini answers this question creating a bridge between the metaphors of mathematics and the ones of music.

This concert/seminar is an introduction to the work of composer and pianist Petra Cini. She is joined by mathematician Raf Bocklandt, who collaborated with the composer on the creation of the (meta-)mathematical framework used for the development of her work SO(3) ETUDES (2021-2023).

Contents

Introduction

Two words: Stark contrasts.

Three elements: Body, mathematical concepts, mysticism.

The structure of this concert/seminar reflects the two different approaches, methods of creation that I employ: a primarily intuitive one and a primarily deductive one.

On the one hand, the highly intuitive approach draws on a focus on the development of a rhythmic and contrapuntal sensitivity, reflected in my choices of repertoire as a pianist. It manifests itself then as this characteristic sensitivity in the composition of, for example, the intuition-driven etudes for piano from the opus ETUDES (2020-) that you will hear in the first half of the programme. Next to these etudes I will perform three counterpoints from the Art of Fugue by J.S. Bach.

On the other hand, the highly deductive approach is concretized in a focus on drawing connections between the metaphors of mathematics and the ones of music. A creation of visceral analogies of mathematical groups, which are abstract algebraic objects. This is achieved by analogically and perceptually analyzing the structure and elements of mathematical groups in terms of violence and purity, which is the fundamental idea of my musical theory. The second half of the evening will be dedicated to this way of working, with the presentation of the opus SO(3) Etudes (2021-2023) and its successive discussion.

I am extremely grateful that I had the opportunity of collaborating with mathematician Raf Bocklandt, who helped me by providing the mathematical insights that were necessary to develop the SO(3) Etudes (2021-2023). Before the performance of the work, Raf will give an engaging talk about symmetry, with a focus on Lie theory and the Lie group SO(3).

I also want to thank the mathematics students from UvA Jelle Groot, Giacomo Grevink, Yiyuan Chen and Meike de Jong for working on the mathematics section of this booklet, and of course Gerard Bouwhuis for hosting this meta-musical evening.

25 March 2023
Petra Cini

Programme

Petra Cini
6 piano etudes from ETUDES (2020-)

J.S. Bach
3 counterpoints from the Art of Fugue

Raf Bocklandt
A short story of symmetry

Break

Petra Cini
SO(3) ETUDES (2021-2023)

Petra Cini and Raf Bocklandt
Meta-mathematical musings

Mathematics

3.1 Some history and intuition

History of groups

In the 19th century a new area in mathematics developed: The theory of groups. Groups are a mathematical way of formalizing symmetry. And since symmetry is used in all branches of mathematics, groups were a very hot area in mathematics when they were invented.

They were first used by the young Frenchman Évariste Galois to prove that a polynomial of degree 5 or higher doesn't have a general formula for its roots based on its coefficients.

Figure 3.1: Évariste Galois. [1]

His ideas, however, were rejected by his peers and his work was only published after his death, which came far too soon for the young Évariste: driven by love for a girl he challenged a romantic rival to a duel, only to be lethally shot. This happened when he was just 20 years of age.

Another mathematical branch where symmetries are very important is geometry. In 1872 Felix Klein published the Erlangen Program, a fundamental work for geometry. This was a method of characterizing different geometries, and for this method he used a lot of group theory. Felix Klein is also well-known for discovering the famous finite group V_4, which is even called the four group of Klein. Klein was very interested in finite groups. [2]

History of Lie

Marius Sophus Lie was a Norwegian Mathematician. He was one of the main collaborators of Group theory. Lie was born on the 17th of December, 1842 in Nordfjordeid. After he graduates from secondary school he keeps studying the sciences and publishes his first mathematical work *Repräsentation der Imaginären der Plangeometrie* in 1869. After this publication Lie receives a travel scholarship to Berlin. In Berlin he meets Felix Klein, a now well known mathematician who is known for his contribution to group theory. Here Lie and Klein become best friends.

Figure 3.2: Sophus Lie

In 1870 Lie goes to Paris where he seeks out for other mathematicians and Klein joins him in April. They speak with French mathematicians Darboux and Jordan. On the 19th of July the Franco-Prussian War breaks out. Because they are in Paris, Klein, a German, has to flee and Lie decides to walk from Paris to Milan. Unfortunately he did not come far. Just outside of Paris he was arrested, because the French thought he was a German spy. After a couple of weeks he is released and he decides to take a train to Milan and then to Düsseldorf, where he meets Klein again.
Eventually Lie and his family move back to Norway where

Lie engages in the Norwegian school and university policy debates. He also publishes his final book *Geometrie der Berührungstransformationen*. Lie receives the Lobachevsky Prize in 1897 for his work in geometry.
[3]

Intuitive definition of Groups and Lie groups

Intuition of Group Theory

Groups are all about symmetries. To understand groups, let us consider an equilateral triangle (all three sides have the same length). When most people think of symmetrical objects, they often referred to them as objects that can be divided in to two identical halves by a mirror line. If you flip the object 180 degrees around the mirror line, you get the same object. In our case of the triangle, we can draw a mirror line through each vertex, labeled with a, b and c. And at each mirror line, we can define a reflection where we flip the triangle around that mirror line by 180 degrees. These reflections are denoted with σ_1, σ_2 and σ_3 (see figure 3.3).

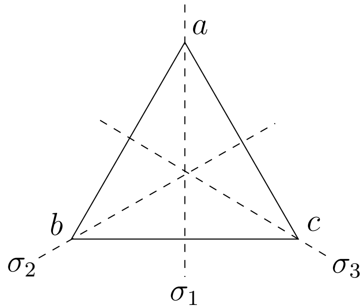

Figure 3.3: Reflective symmetry in an equilateral triangle.

To mathematicians, there are more symmetries hidden in this object. They regard symmetries in a broader and more abstract sense:

> *A symmetry is some transformation that leaves the orientation of an object unchanged.*

The earlier mentioned reflections σ_1, σ_2 and σ_3 are symmetries by this definition. They are called reflective symmetries. However, the most trivial example is "do nothing", since the orientation of the object is

always unchanged if we do nothing to the object. Mathematicians denote this transformation with e, and gave it a fancy name, *the identity*.
Another example is rotational symmetry. We can rotate the triangle clockwise by 120, 240 and 360 degrees, denoted with ρ_{120}, ρ_{240} and ρ_{360} respectively, and the orientation of this triangle would remain the same (see figure 3.4). If we label the vertices of this triangle with a, b and c, we will notice that a 360 degrees rotation has the same effect as "do nothing".

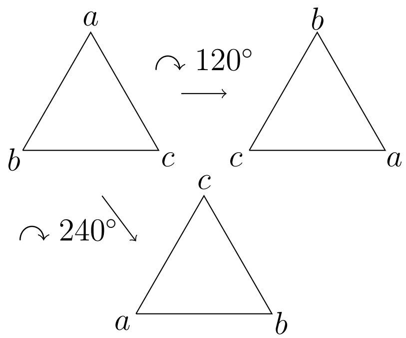

Figure 3.4: Rotational symmetries of an equilateral triangle.

We can now put all these transformations together in a collection,

$$G = \{e, \rho_{120}, \rho_{240}, \sigma_1, \sigma_2, \sigma_3\}.$$

In this collection G, we ignored ρ_{360} because it is the same as the identity e. Now, you might wonder if G contains all the symmetries of the triangle we can make from rotations and reflections. What about the combinations of these transformation?
In fact, the special thing about G is that every combination of transformations in G is again another transformation in G. For instance, when we apply a 120 degree rotation first, and then a 240 degree rotation, we denote this combination as

$$\rho_{240} \circ \rho_{120}.$$

This is the same as ρ_{360}, or "do nothing". And it is not difficult to see that any combinations or rotations leaves us with another rotation. Similarly, the combination of two reflections also delivers a rotation. For instance,

$$\sigma_3 \circ \sigma_1 = \rho_{240}.$$

Last but not least, the combination of a rotation and a reflection gives another reflection. For example,

$$\rho_{120} \circ \sigma_3 = \sigma_2.$$

The verification of the last two statements is left to the reader as an exercise.
As you might have guessed, G is what we call a $group$. In short, a $group$ is a collection of transformations, such that any combination of transformations from this collections gives us another transformation in this collection. Since there is only a finite amount of elements in G, we call G a finite group.
However, there are also groups that contain infinitely many and "continuous" transformations. For example, a circle has infinitely many and continuous rotational transformations, since we can rotate the circle by any degree between -180 and 180, and this rotation will leave the orientation of the circle unchanged. In fact, this is an example of Lie Groups, called $SO(2)$.

3.2 A Geometrical Interpretation of Lie Groups

Lie group $SO(2)$

As discussed earlier, $SO(2)$ is an example of a Lie group. It represents all rotations of a circle. Each rotation of a circle is entirely defined by its angle, which lies between -180 and 180 degrees. A very interesting property of Lie groups is that they can be visualized as a geometric structure. In the case of $SO(2)$, this geometric structure is a circle. We can identify each point of this circle as the angle between that point, and the center of the circle. So $SO(2)$ represents all rotations of a circle, but it is itself a circle as well! This can cause some confusion. One has to remember that the circle that we want to rotate, and $SO(2)$ are not the same circle! A rotation of the circle that we want to rotate corresponds with a point on the circle that is $SO(2)$.

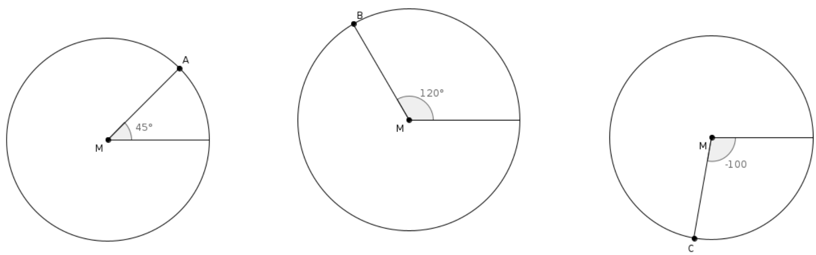

Figure 3.5: The points A, B and C on the circle represent the rotations with 45, 120 and -100 degrees respectively.

Lie group $SO(3)$

$SO(3)$ is the Lie group that represents all the rotations of a sphere. Such a rotation can be visualized as follows: slice the sphere into two equal parts through the center of the sphere. The surface along which the sphere has been cut is a circle. The rotation of the sphere is actually just a rotation of this circle. Using $SO(2)$, we know how to rotate a circle! We do have different choices for which circle we want to rotate, depending on the way in which we sliced the sphere.

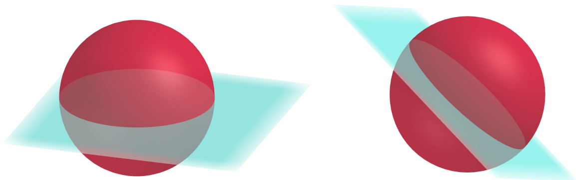

Figure 3.6: Two different ways in which a sphere can be sliced, once along the equator and once along a 45 degree angle.

Again, $SO(3)$ is a Lie group, so it can be visualized as a geometric structure. In this case, the geometric structure is a sphere. How does a point in a sphere correspond with a rotation of a sphere? Start by drawing a line from the point in question to the center, and extend the line to the edges of the sphere. So we want this line to go from the edge of the sphere, through the center and the point we are interested in, and onto the other edge of the sphere. Now, the angle of this line corresponds with how we slice the sphere, and so it corresponds with the angle along which we perform a rotation in $SO(2)$. (This angle is two-dimensional, but we don't have to worry about that, it all works out). So using the angle of the point we know where we need to perform a $SO(2)$ rotation, but we do not yet know how large this rotation will be. As we know from before, this can be anything between -180 and 180

degrees. This will be determined by how far our point lies from the center of the sphere. If the point is all the way on one side, it will be a rotation of -180 degrees, if it is in the center, it will be a rotation of 0 degrees and if it is all the way on the other edge of the sphere, it will be a rotation of 180 degrees. Using this method, each point in the sphere indeed corresponds with a rotation of the sphere. However, the sphere that we have found does have an odd property. We know that a rotation of 180 degrees and a rotation of -180 degrees is the same. So, this means that two points that lie exactly opposite each other on the edge of the sphere, represent the exact same rotation! So if we want to see $SO(3)$ as a sphere, we need to identify opposite points on the edge. This is very hard to visualize.
So $SO(3)$ represents all rotations of a sphere, and is itself a sphere, but a very weird sphere. Again, the sphere that we want to rotate, and $SO(3)$ are not the same sphere. A rotation of sphere we want to rotate corresponds with a point in $SO(3)$. Furthermore, the sphere that we want to rotate is a perfectly normal sphere, while $SO(3)$ is apparently not. So the spheres in figure 3.6 are the sphere that we want to rotate, while the sphere in figure 3.7 is $SO(3)$.

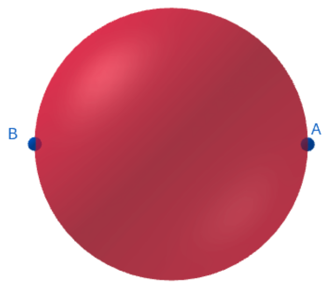

Figure 3.7: As A and B lie on the edge and exactly opposite each other, they represent the same rotation, so we should see them as being the same point in $SO(3)$!

Lie Algebra:

Each Lie group has a corresponding Lie algebra. The Lie algebra can be seen as a representation of the Lie group at a very basic level: it can tell you some basic properties of the group. This can be visualized as zooming into the group, and looking at very small changes between elements of the group, and how those changes behave. Whilst zooming into the group, some information of the group is lost. This can cause two different Lie groups to have the same Lie algebra. For example, we can look at a Lie group that is represented by a sphere, $SO(3)$, and a Lie group that is represented by a doughnut. If we were to zoom in into those groups to find their respective Lie algebra's, at some point, we won't be able to distinguish the two groups anymore, as very small surfaces of spheres and doughnuts are basically the same. So different Lie groups can share a Lie algebra.

About

Petra Cini

is an Italian composer and pianist based in the Hague (NL) focusing on the creation of musical representations of mathematical groups. She is also devoted to the development of a rhythmic and contrapuntal sensitivity, reflected in her choices of repertoire as a pianist.

Petra is currently pursuing a Master's Degree in Composition from the Royal Conservatoire The Hague for which she was awarded an Excellence Scholarship. Her master research project 'The Musical Metaphor and Representation Theory' was awarded funding by Stichting De Zaaier. Previously, she studied piano with pianist Gloria D'Atri, among others, and at the International Piano Academy "Incontri col Maestro" in Imola with internationally known pianist Jin Ju. She holds a Master's Degree in Piano Performance from the Conservatory "Luigi Cherubini" (10/10). In collaboration with Prof. Raf Bocklandt and Prof. Eric Opdam from the Korteweg-de Vries Institute for Mathematics, University of Amsterdam, she is now developing concert/seminars focusing on musical representations of Lie groups.

Further work:

Youtube: https://www.youtube.com/@PetraCini

Website: http://www.petracini.it/

Theory: http://www.petracini.it/works/a-damaged-purity/
https://www.youtube.com/playlist?list=PLZAsHf-N7B_q__VXQO92uSqWbcDJUlcYl

Raf Bocklandt
is a mathematician at the KdVI at the University of Amsterdam specializing in representations of quivers and its applications.

Raf Bocklandt was born in Hamme in Belgium. He studied mathematics at the university of Ghent and did a Phd at the University of Antwerp. After doing Post-docs in Bielefeld and Rome, he became a lecturer at the University of Newcastle. In 2013 he moved to the University of Amsterdam and since 2021 he is the program director of the Bachelor Mathematics. Raf is interested in many aspects of Geometry and Algebra, but he is also involved in science popularizing. In 2019 he gave a public lecture for the Universiteit van Nederland about geometry of higher dimensional spaces.

More info:

Youtube: https://www.youtube.com/watch?v=1zVAPVUHrF0&t=191s

Website: http://algebra.hopto.org/wis/website/

KdVI website: https://kdvi.uva.nl/?cb

Credits

Location:

Splendor Amsterdam

Booklet made by:

- Jelle Groot
- Giacomo Grevink
- Yiyuan Chen
- Meike de Jong

Supervised by:

- Petra Cini
- Raf Bocklandt

Bibliography

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[3] Arild Stubhaug. *The mathematician Sophus Lie : it was the audacity of my thinking*. eng. Berlin [etc: Springer, 2002. ISBN: 3540421378.